

# THE EVOLUTION OF PROTOPLANETARY DISKS AROUND MILLISECOND PULSARS: THE PSR 1257 +12 SYSTEM

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## ABSTRACT

We model the evolution of protoplanetary disks surrounding millisecond pulsars, using PSR 1257+12 as a test case. Initial conditions were chosen to correspond to initial angular momenta expected for supernova-fallback disks and disks formed from the tidal disruption of a companion star. Models were run under two models for the viscous evolution of disks: fully viscous and layered accretion disk models. Supernova-fallback disks result in a distribution of solids confined to within 1-2 AU and produce the requisite material to form the three known planets surrounding PSR 1257+12. Tidal disruption disks tend to slightly underproduce solids interior to 1 AU, required for forming the pulsar planets, while overproducing the amount of solids where no body, lunar mass or greater, exists. Disks evolving under 'layered' accretion spread somewhat less and deposit a higher column density of solids into the disk. In all cases, circumpulsar gas dissipates on  $\lesssim 10^5$  year timescales, making formation of gas giant planets highly unlikely.

*Subject headings:* planet formation: general — pulsars: general, planet formation: individual (PSR 1257 +12)

## 1. INTRODUCTION

The planetary system orbiting the pulsar B1257+12 first reported in 1992, (Wolszczan & Frail, 1992) and subsequently confirmed (Wolszczan, 1994 & 2000), was the first extrasolar planetary system discovered. By the time of the discovery of the first planet around a solar-type star (Mayor & Queloz 1995; Marcy & Butler 1996) the planetary nature of the system had already been confirmed and elucidated by the detection of the signal of resonant perturbations between the two largest planets in the system (Wolszczan 1994). The eccentricities of all three known planets are now known to be extremely low ( $e < 0.026$ ), and the orbits of the outer two planets are nearly coplanar (relative inclination  $< 6^\circ$ ) (Konacki & Wolszczan, 2003, hereafter KW03). Table 1 lists the orbital parameters the three planets (from Table 2 of KW03). Wolszczan (1996) has also argued for the presence of a fourth signature from the pulsar timing data, though its mass and semimajor axis are uncertain.

However, despite this conspicuous head start, there is still very little attention devoted to the theory of planet formation in systems of this type. Such neglect is no doubt due to the fact that the initial conditions for planet formation around a pulsar must necessarily be very different than in the case usually considered in planetary formation theories. In particular, the protoplanetary disk in this case represents an adjunct to stellar death, rather than stellar birth. It is important to note, however, that the very different provenances need not, in fact, give rise to very different conditions. Most theories for how a millisecond pulsar might acquire a protoplanetary disk (reviewed in Podsiadlowski 1993) result in a gaseous disk in orbit about a central gravitating body of

roughly  $1M_\odot$ . Most such theories even describe a disk of approximately solar composition (although some, such as that of Livio, Pringle & Saffer 1992, produce a disk very poor in hydrogen and helium).

This situation is sufficiently similar to the standard picture that it may be analysed with the same tools as are normally used in the study of planet formation. Of course, there are still some fundamental differences between the situation we discuss here and the traditional one. Most importantly, the planets are formed from an expanding disk since the original extent of the disk is  $\lesssim$  several AU (the justification for this is described in Section 2). Only that material which expands out (as the result of viscous evolution) to the distances of the current planets is important for planet formation. This is different from the normal case, where much of the mass is added to the disk at large radii as the result of accretion from a protostellar cloud. Nevertheless, the same fundamental physics behind viscous protoplanetary disk evolution should still apply.

In this paper we investigate the evolution of the gaseous protoplanetary disk around a pulsar, incorporating a detailed description of the physical state of the gas and how this affects the disk evolution. This can be regarded as an extension of Phinney & Hansen (1993) and Hansen (2000), using more detailed models and incorporating modifications to the standard model for the evolution of gaseous proto planetary disks, specifically the "layered disk" model (Gammie 1996; Armitage, Livio & Pringle 2001).

The structure of our paper is as follows. In Section 2 we describe the two likely scenarios for producing protoplanetary disks surrounding millisecond pulsars, the supernova fallback and tidal-disruption scenarios, and discuss the initial disk conditions that result from them. In Section 3 we discuss the standard model for viscous protoplanetary disk evolution as well as modifications to it, incorporating the layered disk accretion theory. We describe the numerical procedure and initial conditions for

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our model runs in Section 4. Section 5 details the gas and dust column density ( $\Sigma_g$  &  $\Sigma_d$ ) and disk central temperature ( $T_c$ ) evolution assuming a supernova-fallback disk, comparing results for a fully viscous disk model with a layered disk model. Section 6 presents results for the disk evolution assuming a tidal-disruption disk. In Section 7, we summarize our results describe future work to model the formation of planets around millisecond pulsars.

## 2. SOURCES FOR PROTOPLANETARY DISKS AROUND MILLISECOND PULSARS

Two leading formation scenarios for protoplanetary disks around pulsars are now investigated: the supernova-fallback and tidal disruption scenarios. These models span a wide range of initial disk angular momenta:  $\sim 10^{49}$ - $10^{52}$  ergs  $\cdot$  s.

### 2.1. Supernova-Fallback Disks

The simplest model for the formation of a protoplanetary disk around a pulsar is the supernova-fallback scenario whereby some fraction of the outer shell mass of the supernova fails to escape the gravitational potential of the surviving neutron star. This material then settles into a disk surrounding the pulsar. Chevalier (1989), Hashimoto et al. (1989) and Lin et al. (1991) modeled supernova fallback, arguing that  $\lesssim 0.1 M_\odot$  of material may remain. The initial angular momentum for such a disk is rather low, typically  $\sim 10^{49}$  ergs  $\cdot$  s (Menou et al. 2001).

Similarity solutions used to describe the structure of a fallback disk were derived by Menou et al. (2001) and were used to argue for confinement of the disk to sub-AU scales ( $\sim 10^{12}$  cm). However, these solutions assume a balance between heating and cooling in determining the temperature structure of fallback disks, an assumption generally more appropriate for disks with far larger initial angular momenta. We take a different approach, allowing for imbalance of heating and cooling as well as advection in determining the disk temperature evolution. The fallback material is assumed to be metal-rich, and the total mass is set at  $\sim 10^{-3} M_\odot$ , consistent with the fallback mass of Type II supernovae (Chevalier 1989).

### 2.2. Tidal-disruption Disks

Alternatively, the disk may form from interactions with a binary companion. We base our assumption for the initial distribution of disk material on the initial angular momentum expected from scenario 5 in Phinney & Hansen (1993): the circumpulsar disk material originates from a companion star that was tidally disrupted after the supernova of the primary. This would occur if the off-center supernova recoil either kicked it into or close to a binary companion or the post-SN orbital parameters for the secondary resulted in tidal disruption.

There are some indications that tidal disruption can be a possible explanation for the origin of a circumpulsar disk. An analysis of post-SN orbital parameters from Kalogera (1996) demonstrate that the survival rate of post-SN binaries is much greater for small initial orbital separations. Post-SN parameters favor a binary separation equal to or lesser than the initial separation and also favor more highly eccentric orbits, factors which make tidal disruption possible. If tidal disruption occurred as the pulsar kicked into its companion at a high velocity

then such a pulsar would likely have a high proper motion. The PSR 1257+12 system exhibits an extremely high space velocity of  $\sim 300$  km  $s^{-1}$ , consistent with this disk formation scenario. The resulting circumpulsar disk would have an initial angular momentum  $J \sim 10^{51}$ – $10^{52}$  ergs  $\cdot$  s. The disk composition is assumed to be solar.

## 3. VISCOUS DISC SURFACE DENSITY AND TEMPERATURE EVOLUTION

Our models begin with an initial protoplanetary disk confined to small radii ( $\leq 3AU$ ), which should be true of both scenarios for disk formation (e.g. Phinney & Hansen 1993). Viscous evolution in this disk results in the accretion of most of the material, but some of the mass must flow outwards to conserve angular momentum (Lynden-Bell & Pringle 1974). Eventually some of this material condenses out into solids and may thus provide the seed material for the observed planets. The goal of our calculations is to examine whether there is sufficient material in the disk to plausibly explain the observed planetary mass. An additional constraint is whether the conditions in the disk are appropriate for the incorporation of dust and solids into planetesimals that would decouple from the gas and begin the process of planet accumulation. This is primarily a temperature threshold – if the gas temperature is too high most heavy elements will remain in a gaseous form.

Phinney & Hansen (1993) examined these issues with simple  $\alpha$ -disk models for a range of initial angular momenta. The results were inconclusive because of two competing effects. For high angular momentum budgets there was indeed enough mass on large scales, but it remained too hot to plausibly deposit solid material until the disk had expanded to scales of several AU, somewhat larger than the observed planetary system. On the other hand, lower angular momentum budgets resulted in the gas becoming cool and depositing mass on smaller scales – too small to really explain the observed planets. Hansen (2000) observed that a potential solution to this problem was to modify the disk evolution to take account of a viscosity that decreases at low temperature – a hypothesis consistent with many treatments of traditional protoplanetary nebulae (Gammie 1996; Sano & Miyama 1999). We now investigate this idea in more detail.

### 3.1. Standard Model

Since the initial disk extent was probably small ( $\lesssim$  several AU) we model the viscous disk evolution using a full diffusion prescription instead of calculating similarity solutions as in Menou et al. (2001). Under the thin disk approximation, the viscous evolution of the surface density  $\Sigma$  is given by the following equation (e.g. Pringle 1981; Hartmann 1998):

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[ r^{1/2} \frac{\partial}{\partial r} (\nu \Sigma r^{1/2}) \right], \quad (1)$$

where  $\nu$  is the viscosity. This viscosity is determined by the microphysics of the gas and is affected by the midplane temperature. The viscosity is parameterized as an  $\alpha$  viscosity (Shakura and Sunyaev, 1973)

$$\nu = \alpha c_s^2 / \Omega \quad (2)$$

where  $\alpha$ , a constant which relates the efficiency of viscous transport, is set to  $10^{-2}$  for MHD-driven turbulent flow.

Viscous heating, irradiation, radiative cooling, and radial advection of material through the disk determine the disk temperature evolution

$$\frac{\partial T_c}{\partial t} = \frac{2}{\Sigma c_p} (Q_+ + Q_{irr} - Q_-) - v_r \frac{\partial T_c}{\partial r} \quad (3)$$

where  $c_p$  is the disc specific heat ( $c_p \sim 2.7 \frac{\mathcal{R}}{\mu}$ ),  $\mathcal{R}$  is the gas constant, and  $\mu \sim 2.3$  is the mean molecular weight for temperatures  $\sim 10^3$  K.  $Q_+$  is the viscous heating rate given as

$$Q_+ = \frac{9}{8} \nu \Sigma \Omega^2, \quad (4)$$

and  $Q_{irr}$  is the heating rate from irradiation (due to accretion onto the pulsar)

$$Q_{irr} = \frac{L_{acc}}{4\pi r^2} (1 - \beta) \cos \phi, \quad (5)$$

where  $\cos \phi = \frac{dH}{dr} - \frac{H}{r}$  and the albedo ( $\beta$ ) is 0.5 (Frank, King, and Raine 2002; see also King and Ritter 1998). We assume that  $L_{acc}$  is the minimum of the accretion luminosity onto the pulsar and the Eddington luminosity, and allow for self-shadowing of outer regions of the disk by puffed-up inner regions.  $Q_-$  is the radiative cooling term given as

$$Q_- = \sigma T_e^4, \quad (6)$$

and the vertically averaged radial velocity of material through a cross section in the disc is

$$v_r = -\frac{3}{\Sigma r^{1/2}} \frac{\partial}{\partial r} (\nu \Sigma r^{1/2}). \quad (7)$$

The cooling rate can be cast in terms of the midplane temperature  $T_c$  by relating  $T_c$  to the local effective temperature ( $T_e$ ) through the condition of vertical radiative transport. From Hubeny (1990) we find that in the optically thick limit

$$T^4 = \frac{3}{8} \tau T_e^4 \quad (8)$$

where  $\tau = \frac{\Sigma}{2} \kappa$  and for the optically thin limit

$$T^4 \sim T_e^4 / \tau. \quad (9)$$

To complete the microphysical description of the gas, we compute the opacity  $\kappa$  as a function of temperature and density. We use the frequency-averaged opacities from Bell & Lin (1994), shown in Table 2 with slight modifications. The analytical fits to these opacities have the form of  $\kappa = \kappa_o \rho^a T^b$ . The temperature range over which a given analytical fit of  $\kappa$  is valid is determined by requiring continuity of the function  $\kappa$  between different domains. For low temperatures, the temperature range over which a given power-law prescription is valid is independent of density. When the disc is partially ionized there is a non-negligible dependence on the density and thus  $\Sigma$  for the boundaries:

$$T_{max,n} = \left[ \frac{\kappa_n}{\kappa_{n+1}} \left( \Sigma \Omega \left( \frac{\mathcal{R}}{\mu} \right)^{1/2} \right)^{a_n - a_{n+1}} \right]^{\frac{1}{b_{n+1} - b_n + 1/2(a_n - a_{n+1})}} \quad (10)$$

where the midplane gas density is given as  $\rho_c = \Sigma c_s / \Omega$  and the adiabatic sound speed is given as  $c_s = \sqrt{\mathcal{R} T_c / \mu}$ . The opacities in Table 2 assume solar abundance. Menou et al. (2001) note that opacities for species expected in

supernova-fallback disks are similar to those for a solar mixture.

The analytical fits from Bell & Lin (1994) compare favorably with more sophisticated numerical calculations (e.g. Semenov et al. 2003; Ferguson et al. 2005) over most temperature ranges. However, they underpredict the opacity by as much as three orders of magnitude in the  $n=4-5$  regimes as can be seen from Semenov et al. (2003; at  $\log(T) \sim 3.25$ ). By the  $n=6$  ( $\kappa \propto T^{10}$ ,  $\sim \log(T)=3.5$ ) regime, the opacities are again consistent with other calculations. Semenov et al. cite an improper truncation of the molecular water opacity as the likely source for this discrepancy. Because numerical calculations from several different groups (e.g. Pollack et al. 1994; Alexander & Ferguson 1994; Semenov et al. 2003) all find a much higher opacity, there is strong evidence that the Bell & Lin opacities are too low. Therefore, when a disk region is in the  $n=4-5$  opacity regimes we set a 'floor' to the opacity equal to opacity at transition temperature between  $n=4$  and 5, where the Bell & Lin predictions are consistent with others. Thus, over a small temperature range the opacity is roughly constant and comparable to the values of Semenov et al. and others.

### 3.2. Modifications for Layered Disk Evolution

Extensive work over the last decade has shown the effectiveness of the magnetorotational instability (MRI) in providing the anomalous viscosity for accretion disks in a variety of astrophysical situations (Balbus & Hawley 1991). However, one situation where its usefulness is debatable is precisely the one of interest in this case – a protoplanetary disk. This may be especially true in the SN fallback case where the cooling disk may quickly become neutral and thus unable to drive MHD turbulence (Lin et al. 1991; Menou et al. 2001). The principal issue is the ionization state of the gas which is an exponential function of the gas temperature (see Gammie 1996). When the disk gas is only partially ionized, the coupling between magnetic field and disk gas is weakened and eventually the MRI ceases to operate. The gas temperatures in the outer parts of protoplanetary disks are perhaps a few hundred Kelvin, so that ionization is expected to be very low and the viscosity may be considerably reduced.

This requires that we modify the disk evolution described in Section 3.1. In particular, we adopt the formalism of Gammie (1996), in which accretion is assumed to occur only through a thin 'active' layer on the surface of the disk. Ionization is maintained by irradiation, either by cosmic rays or X-rays from the central star, and so the MRI can still operate. The rest of the material is assigned to an inviscid, unevolving 'dead' layer. Thus, at each radius we must consider both an active surface density  $\Sigma_a$  as well as a total surface density  $\Sigma$ . The relative extent of the active layer is a function of the magnetic Reynolds number in the midplane (Fleming & Stone 2003), with the transition occurring over a finite temperature range.

The ionization fraction for solar abundances is dominated by collisionally-ionized potassium (Umebayashi 1983) and varies over five orders of magnitude from 1000K to 800K. Thus, for tidal disruption disks we assume that the disk is fully viscous at 1000K and transi-

tions (linear in  $\log T$  space) to a layered state by 800K. The treatment for the supernova-fallback disk is more difficult since such disks are unlikely to have a solar composition of elements and also may be initially hot enough to trigger nucleosynthesis. To address this issue, we follow Fujimoto et al. (2001) who used the pre-SN chemical composition models of a  $20 M_{\odot}$  progenitor from Nomoto & Hashimoto (1988) to model the initial conditions and early evolution of fallback disks. Nucleosynthesis in the disk is possible at stellocentric distances  $\leq 3 \times 10^3 r_g$ , where  $r_g$  is the Schwarzschild radius ( $\frac{2GM}{c^2}$ ), which corresponds to  $\sim 2 \times 10^{10}$  cm for a  $20 M_{\odot}$  progenitor. This distance is probably too small to be relevant for planet formation from fallback disks. Regions beyond this should have a similar composition to that of the ejected layers of the pre-SN star. The exact composition of these layers varies strongly. Fujimoto et al. considers three layers: a silicon rich, oxygen rich, and helium-rich layer. We assume that these layers are well mixed in the disk and calculate the mean composition of the disk assuming that all layers contribute equally. The dominant gas-phase elements are O, Si, He, and S;  $\sim 26\%$  are metals. Because a sizeable fraction of metals are retained (including, presumably, potassium) we assume that the disk transitions from fully viscous to layered over the same 1000 to 800K range.

In both the fallback and tidal-disruption disk models, we assume that  $\Sigma_a$  has a minimum of  $200 \text{ g cm}^{-2}$  which is twice the stopping surface density for cosmic rays using interstellar medium values ( $100 \text{ g cm}^{-2}$  for two disk faces). The stopping depth for x-rays is a much smaller  $\sim 0.1 \text{ g cm}^{-2}$ .

Thus, for layered accretion equations 1, 3, and 6 are now modified such that

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} [r^{1/2} \frac{\partial}{\partial r} (\nu \Sigma_a r^{1/2})], \quad (11)$$

$$Q_+ = \frac{9}{8} \nu \Sigma_a \Omega^2 \quad (12)$$

and

$$v_r = -\frac{3}{\Sigma_a r^{1/2}} \frac{\partial}{\partial r} (\nu \Sigma_a r^{1/2}). \quad (13)$$

The vertically averaged radial velocity of the disc,  $v_r$ , becomes smaller as the disc evolves into a layered state since the energy transported by the disk active layers is shared by the entire disk bringing the dead zone into thermal equilibrium with the base of the active layer. The temperature change due to advection is then reduced by  $\Sigma_a/\Sigma$  in the layered disk region. The optical depth,  $\tau$ , is now given as  $\tau = \frac{\Sigma_a}{2} \kappa$ .

The small  $\Sigma_a$  value may underestimate the true column density of material transported. The cosmic-ray flux at the center of a supernova remnant may in fact be significantly larger than typical interstellar medium values. Furthermore, while most of the disk may be magnetically dead, it is not clear that the same column of material is mechanically dead: turbulent eddies in the active region may overshoot the active/dead boundary and transport some mass from the dead zone (see Fleming & Stone 2003). Therefore, the fully viscous and layered prescriptions adopted here should be seen limiting cases for viscous transport in circumpulsar disks, where the viscous evolution actually manifest in circumpulsar disks is likely somewhere in between.

### 3.3. Solid Body Evolution

Modeling only the gas evolution is not sufficient for investigating possible planet formation out of such a disk as a fraction of the original disk mass will condense into solids once the disk cools past certain temperature thresholds. Here we describe a crude model for tracking the solid body evolution of an originally all-gaseous circumpulsar disk. We assume that once the disk at some stellocentric distance drops below a temperature threshold it removes a fraction of its mass to solids in the next timestep. This neglects dust dynamics in the evolution of the circumpulsar disk. Specifically, we first have neglected the time required for grains to settle to the disk midplane. As this depends critically on the grain size as well as the level of anisotropy of disk turbulence - the component of turbulence acting vertically must be extremely low for complete settling (e.g. Dubrulle et al. 1995) - we make the simplifying assumption that the solid bodies can settle to the disk midplane on short timescales and that turbulence is unable to stir particles to large scale heights. Highly anisotropic hydrodynamical forms of turbulence (e.g. Youdin & Chiang 2004) or layered disk models operating under MRI turbulence (e.g. Currie 2005) should allow grains to settle to a thin scale height. Second, we neglect the effect of radial migration. Such migration may alter the dust-to-gas ratio in disks, trigger planetesimal formation (Youdin & Shu 2002), and perhaps even alter the efficiency of planet formation (Currie 2005). However, as we shall see, gas from circumpulsar disks typically depletes on very short,  $\sim 10^5$  year timescales. Furthermore, as the disk cools very large columns of dust, much greater than the Minimum Mass Solar Nebula (MMSN; Hayashi 1981) may be deposited for some models. Both of these conditions hasten planetesimal formation by gravitational instability such that grain radial migration may not be a strong effect. Therefore, while simple, this model should still allow one to get a zeroth-order understanding of the expected regions of planet formation from a circumpulsar disk.

The first temperature threshold we use occurs at  $\sim 500\text{K}$  which takes into account the condensation of metal grains which should finish around  $\sim 470\text{K}$ . Once a gridpoint drops below  $500\text{K}$  we remove a fraction of its mass from the gas disk and place it into a solid body distribution:

$$\Sigma_{d, \text{metals}} = c \times \Sigma_g. \quad (14)$$

Because the supernova fallback disk is comprised of 26% metals,  $c=0.26$  in the supernova fallback model. For the solar-composition tidal disruption model, we set  $c=0.01$ .

The second temperature threshold, relevant for the tidal-disruption model, occurs at the water ice condensation point. This transition occurs roughly at  $170\text{K}$  and removes from the disk a larger fraction of mass than the condensation of metals. Specifically, when a gridpoint drops below  $170\text{K}$  we then remove the following mass from the gas disk and deposit it into the solid body distribution

$$\Sigma_{d, \text{ice}} = 0.032 \times \Sigma_g. \quad (15)$$

These temperatures are significantly less than  $\sim 1000\text{K}$ , below which the disk is likely magnetically dead and unable to drive turbulence in the disk midplane.

Mass can also be added to the solid body distribution by advection of material from a hot region of the disk

across a temperature threshold to a cooler region. We crudely model this by ascribing a certain fraction (0.01 or 0.26 for across the metal grain threshold and 0.032 for the water ice threshold) of disk material designated for solids once said material crosses a threshold. We then set this fraction to be zero for all regions for which the disk is cool enough to allow condensation. If, after a timestep, that grid point to the cooler side of the threshold now has some 'solids' we add that amount to the solid body distribution for that grid point and subtract it from the calculated gas distribution.

We also track the amount of solid material interior to  $1AU$  and exterior to  $1$  ( $2$ )  $AU$  for the supernova fallback model (tidal disruption model). As the three known planets are all within  $1AU$  of the pulsar we want to see how much mass is contained in this region as a function of time. Specifically, we want to see if, in order to form the known planets, we must 'import' material from beyond  $1AU$  or require that they formed much further out and migrated inward. Models for the fourth signature from this system suggest the existence of a Uranian to Jovian mass body with a large semimajor axis ( $\sim 5AU$  or larger). We test how likely such formation would be in a circumpulsar disk in our formation scenario by tracking the mass of the disk exterior to  $2AU$ .

#### 4. NUMERICAL PROCEDURE

We solve for the evolution of the surface density and temperature profiles using an explicit finite difference method over 250 equally spaced radial grid points. The timestep taken is small at the beginning because of limits of the Courant condition for numerical stability, but it is eventually expanded such that we cover a time evolution of the disc over  $10^6$  yrs. To guard against numerical instabilities arising at the boundaries between opacity regions we smooth the values for the opacity at such boundaries to be a composite of the opacity on either side of the boundary.

We follow the evolution of circumpulsar disks after the point at which it has cooled to  $\lesssim 10,000$  K, prior to which it expands rapidly. Formation scenarios for circumpulsar disks typically start with very compact disks ( $\lesssim 0.1$  AU) and  $\sim 10^{49-52}$  ergs.s. Early tests found that starting the tidal-disruption disk runs with material spread over  $\sim 0.1$  AU scales caused the disk to heat up to  $\gtrsim 10^6$  K until it spread to  $\sim$  several AU in less than 1 year, where it cooled to  $\lesssim 10,000$  K. The lower mass of the supernova fallback disk results in smaller, more transient spreading such that a fallback disk cools to  $\lesssim 10,000$  K after spreading for  $< 1$  AU. Thus, we uniformly spread the disk material for our initial surface density profile over  $0.3$  AU ( $3$  AU) for the supernova-fallback (tidal-disruption) models. Initial temperature for the disc is set at  $\sim 3500$  K though we find that for our initial surface density the temperature quickly reaches values independent of the starting condition.

We present results for the supernova-fallback scenario and then the tidal-disruption scenario. For each scenario we do model runs for full viscous disk models and then layered disk models. For the latter models, we use the active/ dead layer surface density ratios from equations 10 and 11. We start with initial angular momentum values of  $10^{49}$ ,  $10^{51}$ , and  $10^{52}$  ergs.s, corresponding to the supernova fallback model and two tidal-disruption models,

respectively. The goal is to run the disc evolution for reasonable initial models to see what constraints can be put on the planet formation from such disks. We compare our results with the masses and positions of the three known planets in PSR 1257+12, and place constraints on the locations and positions of any other planetary mass bodies in the system. Important parameters for these constraints are the evolution of  $\Sigma_g$ , the gas surface density,  $T_c$ , the central temperature, and  $\Sigma_d$ , the solid mass surface density. We also compute the absolute value of the mass accretion rate throughout the disk ( $\dot{M}$ ) and track the Toomre Q parameter ( $Q_{grav}$ ) to see the disk may periodically become gravitationally unstable.

### 5. SUPERNOVA FALLBACK MODELS

#### 5.1. Fully-Viscous Accretion

Figure 1 shows the gas surface density ( $\Sigma_g$ ) and central temperature ( $T_c$ ) profile evolution for the fully-viscous model, assuming that the disk originates as supernova-fallback material. The disk evolves on a brisk  $10^4$  year timescale. By  $\sim 10^5$  years, the surface density of gas drops to below 1/100th that expected for the Minimum Mass Solar Nebula. For comparison, circumstellar dust and gaseous disk components are expected to evolve on longer,  $\sim 10^6 - 10^7$  yr timescales (e.g. Currie et al. 2007a; Zuckerman et al. 1995), after which only gas-poor/free debris disks remain (e.g. Hernandez et al. 2006; Currie et al. 2007b). Circumpulsar disks evolve on shorter timescales largely because the initial angular momentum of a supernova-fallback disk is  $\gtrsim 100$ -1000 times smaller. A high disk temperature, and thus high rate of viscous transport, results from the compact fallback disk. Furthermore, irradiation from accretion onto the neutron star heats up the inner disk regions to very high ( $\gtrsim 10^5$  K) temperatures during the early evolutionary stages ( $t \lesssim 1000$  yr), causing the disk to spread rapidly. Irradiation impacts the disk at very acute angles and the disk cools after it spreads. This allows all but the innermost disk regions to become self shadowed quite easily after  $\sim 10^3$  yr. Viscous heating is then required to maintain high disk temperatures. This results in a fast cooling of the disk from  $10^3$ - $10^5$  years.

Despite the fast gas accretion timescale, the column density of solids,  $\Sigma_d$ , is still large enough to form approximately Earth mass bodies if the final assembly of planets is very efficient, owing to the rapid cooling of disk material to below 500K (Figure 2) and advection across the 500 K threshold. Typical  $\Sigma_d$  values are  $\sim 10$  MMSN values, up to  $\sim 100\times$  MMSN at  $\sim 0.1$ - $0.5$  AU, even though by  $\sim 10^5$  years the *gas* column density is depleted by a factor of  $\gtrsim 1000$ .

In this model, formation of Earth-sized bodies is highly unlikely outside 1 AU, though it may be possible form asteroid-sized bodies at this distance. There are some suggestions that the presence of such a body is consistent with the pulsar frequency derivatives obtained by KW03 as suggested by Konacki & Wolszczan, in prep. About  $8.6 M_\oplus$  of solids exist interior to 1 AU by  $\sim 10^4$  years. Thus, forming the known planets of PSR 1257+12 from this reservoir of solids requires that the accretion process be  $\gtrsim 95\%$  efficient. Since the gas dissipation timescale is fast, the formation of gas giant planets from a supernova fallback disk is unlikely.

Thus, the likely result from this formation scenario and disk model is a system of one or more Earth-mass bodies, orbiting very close to the star with no planets or circumstellar debris beyond  $\sim 1$  AU. In the next section we see if this result for the supernova-fallback model is sensitive to our viscosity prescription: in other words, does the resulting distribution of solid mass differ if the disk spends much of its time in a layered state?

### 5.2. Layered Accretion

When the disk is allowed to evolve in a layered state the resulting evolution timescale is slightly longer than the fully viscous case,  $\sim 10^5$  years (Figure 3). The disk temperature characteristically remains higher than in the fully viscous model through  $\sim 10^4$  years. The  $\Sigma_d$  profile follows a very different pattern, showing pronounced peaks at  $\sim 0.5$  AU and then later at 0.2 AU (Figure 4). The simulation was run several times with slightly varying initial parameters, and the same peak structure was reproduced each time. This peak is due to a pileup of material at the dead zone/outer active zone boundary. If the disk material cools below 500K before gravitational instability is triggered (e.g. Armitage, Livio, & Pringle 2001), then a substantial mass of solids can condense out of the disk. The very small disk angular momentum, high  $\Sigma_d$ , and assumption of layered accretion combine to produce these peaks in the gas, and in turn dust, column densities. The mass of solids deposited within 1 AU of the pulsar can be enormous under the layered accretion assumption. Figure 5 suggests that  $30 M_\oplus$  of solids can condense out of the disk interior to 1 AU, more than enough to form the known planets orbiting PSR 1257+12. Interestingly, in both the fully viscous and layered disk models, most of the solid material sediments out at roughly the same place,  $\sim 0.2$ -0.5 AU, which is conspicuously similar to the current locations of the known pulsar planets. Because a layered disk spreads so slowly, far less solid material is deposited at distances  $\geq 1$  AU (Figure 5, righthand side). Under this model, any large bodies forming at  $\sim 1$  AU must be  $\lesssim 0.1$ -0.5  $M_\oplus$  in mass. In both the inner ( $\leq 1$  AU) and outer ( $\geq 1$  AU) disk regions, the formation of gas giant planets by core accretion is highly unlikely given the very fast gas dissipation timescale (e.g. Figures 3 and 5).

Thus, a supernova-fallback disk evolving by layered accretion will likely produce several high-mass ( $\sim 1$ -10  $M_\oplus$ ) bodies within  $\sim 1$  AU and may form bodies the mass of asteroids or Kuiper belt objects beyond  $\sim 1$  AU. This resulting architecture is consistent with the known planets in the PSR 1257+12 system but inconsistent with the existence any Uranian or Jovian-mass bodies orbiting at large semimajor axes. In the next section we investigate disk evolution with initial conditions set by the tidal disruption models to see if a different disk formation mechanism is also consistent with the PSR 1257+12 system.

## 6. TIDAL DISRUPTION MODELS

### 6.1. Fully-Viscous Accretion

The first notable difference between the supernova fallback and tidal-disruption models is that the latter tend to have much larger disks, owing to their larger initial mass and angular momentum. Figures 6-8 and 9-11 respectively show the results for a fully viscous disk with

$J \sim 10^{51}$  and  $J \sim 10^{52}$  ergs-s of initial angular momentum. In the first case, by  $\sim 10^3 - 10^4$  years, the disk expands to  $\sim 8$  AU. Gas depletes on  $\sim 10^4$  year timescales (Figure 6), and solid body densities comparable to MMSN values emerge by  $\sim 10^4$  years with most of the mass within  $\sim 3$  AU (Figure 7). Initially, the disk has a very high accretion rate of  $\sim 10^{-6} M_\odot \text{ yr}^{-1}$ , similar to that for classical T Tauri stars, but falls by more than two orders of magnitude within  $\sim 10^4$  years. Just over  $8 M_\oplus$  of material, slightly less than needed to form the pulsar planets, resides interior to 1 AU by  $10^5$  years; a large amount  $\sim 150 M_\oplus$  is deposited outside of 2 AU.

A disk with  $J \sim 10^{52}$  ergs-s of initial angular momentum evolves on similar timescales (Figure 9). The  $\Sigma_d$  profile shows that slightly less (more) mass is deposited interior (exterior) to 1 (2) AU than in the  $J \sim 10^{51}$  ergs-s case. This is due to a slightly longer viscous evolution. The solid mass exterior to 2 AU (Figure 11) exhibits a 'sawtooth' like pattern through  $10^4$  years, owing to evaporation of solids as the outer disk heats up and then cools from expansion. The direct heating of the disk from irradiation also contributed to evaporation of solids, especially in the first  $\sim 10^4$  years. The in-situ formation of  $\gtrsim 5 M_\oplus$ -mass planets is still difficult interior to 1 AU. About 300  $M_\oplus$  of solids exist beyond 2 AU by  $\sim 10^5$  years. The formation of gas giants is, again, highly unlikely because circumpulsar gas depletes to below 10  $M_\oplus$  by  $2 \times 10^5$  years.

In general, we find that the amount of solid mass interior to 1 AU is just under 8  $M_\oplus$  and the amount exterior to 2 AU is  $\sim 100$ -300  $M_\oplus$ . It is then plausible that many 1-10  $M_\oplus$  bodies may be able to form beyond  $\sim 1$ -2 AU in the tidal-disruption scenario for circumpulsar disk formation, though this formation. However, since the disk falls below  $\sim 1000$ K everywhere except for the innermost regions by  $\sim 10^3 - 10^4$  years, the resulting solid body profiles (Figure 7,10), and expected locations for planet formation (Figure 8,11), may strongly depend on the fully viscous assumption. We now investigate what happens if we allow the disk to evolve by layered accretion.

### 6.2. Layered Accretion

Differences in how  $\Sigma_g$  and other parameters evolve in the layered (compared to fully viscous) case are more pronounced for tidal-disruption disk models since the higher initial angular momentum and mass of the disk results in a larger difference between the actively accreting column of gas  $\Sigma_a$  and the total column. This is evident in the  $J \sim 10^{51}$  ergs-s case as shown in Figure 12. By  $\sim 500$  years, the disk between 0.5 and 3 AU has evolved into a layered state.  $\Sigma_g$  in the 'layered' region is relatively higher than in the fully viscous case, owing to the pile up of gas onto the layered region/outer fully viscous region boundary that is subsequently redistributed throughout the disk interior to the boundary. Over the next  $\sim 10^5$  years the size of the layered region shrinks as material piles up onto its outer edge, eventually triggering gravitational instability of the disk at this region. The instability, however, is marginal and operates only long enough to redistribute some mass quickly: planet formation by gravitational instability (Boss 2005) is highly unlikely. Both  $\Sigma_g$  and  $T_c$  evolve on  $\sim 10^5$  year timescales. The long  $T_c$  timescale results from a small

advection term (from  $\Sigma_a \ll \Sigma_g$ ) and a near equilibrium that is reached by the heating ( $Q_+$ ) and cooling ( $Q_-$ ) terms in the  $T_c$  evolution equation. It is not until the advection term becomes important again that the imbalance between heating and cooling becomes upset and the central temperature declines substantially.

The mass accretion rate through the disk follows a constant to inverted structure characteristic of layered accretion (Figure 13). While the accretion rate is several orders of magnitude less than in the corresponding fully viscous case it stays above  $\dot{M} \sim 10^{-10} M_\odot \text{yr}^{-1}$  for  $\geq 10^4$  years, or somewhat longer than in the fully viscous model. The resulting  $\Sigma_d$  profile drops from  $\sim 500$  to  $1000 \text{ yr}$ , owing to viscous heating and irradiation. Eventually the disk cools and the final  $\Sigma_d$  value settles to a median density of  $\sim 30\text{--}40 \text{ g cm}^{-2}$  between 1 and 4 AU by  $\sim 10^5$  years (Figure 13). The amount of solid material interior to 1 AU and exterior to 2 AU is set by  $\sim 10^5$  years (Figure 14). The amount of solid material deposited within 1 AU is  $\sim 7 M_\oplus$ , or not quite enough to account for the three known planets; the bulk of the mass is deposited beyond  $\sim 2$  AU. Even though up to  $200 M_\oplus$  of solids exist beyond 2 AU it is highly unlikely that gas giant planets can form around millisecond pulsars since the gas dissipates on  $\sim 10^5$  year timescales.

Layered disk models initially with  $J \sim 10^{52} \text{ ergs-s}$ , though with a larger initial mass than  $J \sim 10^{51} \text{ ergs-s}$  models, evolve on similar ( $\sim 10^5$  year) timescales (Figure 15). The disk stays in a layered state from  $\sim 10^3$  to  $10^5$  years.  $\Sigma_d$  evolves in a similar way to the other layered disk run though the solid column density above  $\sim 10 \text{ g cm}^{-2}$  extends to 6 AU instead of 4 AU as with the previous run. The densities are  $\sim 10\times$  MMSN column densities through 6 AU (Figure 16). This is because the disk, initially  $\sim 0.1 M_\odot$  is confined to a smaller initial radial spread than for the solar nebula. The mass of solids deposited both interior to 1 AU and exterior to 2 AU is higher than in the fully viscous case by about a factor of two (Figure 17). In general, it then appears that layered accretion disks deposit a higher mass of solids into the disk than fully viscous disks. The circumpulsar gas drops below  $\sim 10 M_\oplus$  by  $\sim 10^5$  years, so that no model predicts the presence of circumpulsar gas at ages comparable to those observed around pre-main sequence stars ( $\sim 10^6\text{--}10^7$  years). Thus, the  $\lesssim 10^5$  year gas dissipation timescale in all model runs imposes draconian requirements on the efficiency of gas giant planet formation even for models that swiftly form the cores of gas giant planets (Rafikov 2004; Currie 2005; Alibert et al. 2005). Such planet formation is effectively ruled out for both fallback and tidal disruption circumpulsar disk formation scenarios. Although true gas giants are unlikely to form, there is sufficient mass to allow for the formation of massive rocky or ice giant planets with no gas envelopes. The persistence of circumpulsar gas may also still strongly affect the dynamics of planetesimals in the feeding zones of growing planets (Rafikov 2004; Currie, Kenyon, & Bromley in prep.) and thus affect the final outcome of the pulsar planet formation process.

We can now summarize our expectations for planet formation around millisecond pulsars when the tidal-disruption model is assumed for the circumpulsar disk origin. In most models not quite enough material within

1 AU forms to account for all the planetary mass in the system; planet formation is preferred beyond  $\sim 1$  AU scales. A Uranus-mass body orbiting at large stellocentric distances is consistent with results from pulsar timing methods, but the formation of a body with a much more massive gaseous envelope (e.g. a Jovian planet) from either the supernova-fallback or tidal-disruption scenarios is extremely unlikely.

## 7. SUMMARY & FUTURE WORK

We have modeled the evolution of protoplanetary disks surrounding millisecond pulsars. We investigated two formation scenarios for these disks, supernova fallback and tidal disruption, under two models for the disk viscous evolution: a fully viscous model and layered accretion model.

For the supernova fallback scenario, the gas density drops well below MMSN values (at  $\sim 1$  AU) by  $\sim 10^4$  -  $10^5$  years while by  $\sim 10^5$  years the solid mass is  $\sim 10$ – $100$  times that of the MMSN values within  $\sim 0.5$  AU of the pulsar. The most likely outcome for supernova fallback disks is a system of Earth mass bodies confined to within 1 AU of the pulsar, a system architecture consistent with the PSR 1257+12 system. For the tidal disruption scenario, gas dissipates on  $\lesssim 10^5$  year timescales, and the solid body distribution reaches  $1\text{--}10\times$  MMSN values by  $10^4$  years with little material extending beyond  $\approx 6$  AU. Most of the mass is deposited beyond  $\sim 1\text{--}2$  AU. The most likely outcome from this scenario is a system of Earth-mass bodies between 2 and 6 AU but little material beyond this distance. Not quite enough solid mass exists to form the known planets in the PSR 1257+12 system in situ. While both models can account for the formation of Earth-mass planets somewhere in the disk, the supernova fallback model favors formation of such planets interior to  $\sim 1$  AU, where the tidal-disruption model favors such formation exterior to  $\sim 2$  AU, even though the starting column density of the fallback disk interior to 1 AU is no greater than that for the tidal disruption model. On this basis, the supernova fallback model more plausibly explains the compact planetary system of PSR 1257+12.

There are several issues important to understanding the formation of pulsar planet systems that have been left untreated by this paper and should be the subject of future study. First, our models neglect the migration of solid grains after they have sedimented out in the disk. Global migration of small grains in either a laminar (Youdin & Shu 2002; Currie 2005) or turbulent (Stepinski & Valageas 1997) can result in a systematic redistribution of solids in a disk, affecting the preferred locations for planetesimal and, eventually, planet formation. While the gas disk in the supernova fallback model evolved on timescales much faster than the migration timescale for grains in the solar nebula (e.g.  $10^4$  yr vs.  $\sim 10^5\text{--}10^6$  yr from Youdin & Shu), tracking the evolution of the solids (e.g. the method of Alexander & Armitage (2007)) should be included to more accurately model the evolution of the solid body distribution once the disk cools enough to allow condensation. Second, given the rarity of planets around millisecond pulsars, the formation of circumpulsar disks from which planets can form should be rather atypical. While disks from the tidal-disruption scenario are obviously rare, the

TABLE 1  
ORBITAL PARAMETERS FOR THE THREE CONFIRMED PLANETS IN  
THE PSR 1257+12 SYSTEM

<i>Planet</i>	$M/(M_{\oplus})$	$r/1\text{AU}$	<i>Period</i> (d)	$e$	$i$ (both solutions)	$\omega$ (deg)
<i>A</i>	0.02	0.19	25.262	0	...	0.0
<i>B</i>	4.3	0.36	66.5419	0.0186	53, 127	250.4
<i>C</i>	3.9	0.46	98.2114	0.0252	47, 133	108.3

conditions for the formation of supernova fallback disks capable of forming planets should be investigated more thoroughly. Third, more detailed pulsar timing data to constrain the nature of the fourth body will help to discriminate between the rival scenarios for forming pulsar planets. The fallback disk scenario would predict that any fourth body at larger semimajor axes is small. A more massive body,  $\sim 1 M_{\oplus}$  or greater, would be more consistent with the tidal disruption model.

Finally, a major issue neglected in this paper is how planet formation would actually proceed in a circumpulsar disk. The key issues with this are whether or not the final (post-oligarchic) stages of pulsar planet formation are characterized by growth (by mergers) like the terrestrial planets, how much material is needed to form the known planets, and how long the final assembly of planets takes to complete. For the solar nebula, the final stages of planet formation interior to  $\sim 2$  AU is primarily characterized by repeated mergers of sub-Earth mass 'oligarchs' (e.g. Kenyon & Bromley 2006). Whether or not the final stage of planet formation from supernova fallback disks is characterized by mergers or ejections of these lunar-mass oligarchs depends on escape

velocity from their surfaces compared to the escape velocity from the system (Goldreich et al. 2004). The age of PSR 1257+12 also sets some limit on the timescale for planet formation. The PSR 1257+12 system is probably  $\sim P/2\dot{P} \sim 8 \times 10^8$  years old (e.g. Miller & Hamilton 2001). Therefore, the merger timescale must be less than  $\sim 10^9$  years. This timescale is somewhat larger than the timescale for merger and cleanup of bodies in the terrestrial zone of our solar system (Goldreich et al. 2004). The merger timescale will depend critically on the amount of solid mass,  $\Sigma_d$ , the final planetary mass,  $M_p$ , and the orbital frequency. Numerical simulations of pulsar planet formation will help to determine the minimum  $\Sigma_d$  allowable, and thus constrain the initial disk conditions from which these planets form. We leave such simulations to future studies.

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#### REFERENCES

- Alexander, D., Ferguson, J., 1994, *ApJ*, 437, 879  
 Alexander, R., Armitage, P., 2007, *MNRAS*, 375, 500  
 Alibert, Y., et al., 2005, *ApJ*, 626, 57  
 Armitage, P., Livio, M., & Pringle, J., 2001, *MNRAS*, 325, 705  
 Balbus, S., Hawley, J., 1991, *ApJ*, 376, 214  
 Bell, K. R. & Lin, D.N.C. 1994, *ApJ*, 427, 987  
 Boss, A., 2005, *ApJ*, 629, 535  
 Chevalier, R., 1989, *ApJ*, 346, 847  
 Currie, T., 2005, *ApJ*, 629, 549  
 Currie, T., et al., 2007(a), *ApJ*, 659, 599  
 Currie, T., et al., 2007(b), *ApJ* accepted, arXiv:0706.0535  
 Dubrulle, B., et al., 1995, *Icar*, 114, 237  
 Ferguson, J., et al., 2005, *ApJ*, 623, 585  
 Fleming, T. & Stone, J. 2003, *ApJ*, 585, 908  
 Frank, J., King, A., & Raine, D., 2002, *Accretion Power in Astrophysics* (Cambridge: Cambridge University Press)  
 Fujimoto, S., et al., 2001, *PASJ*, 53, 509  
 Gammie, C., 1996, *ApJ*, 1996, *ApJ*, 457, 355  
 Goldreich, P., Sari, R., & Lithwick, Y., 2004, *ARA&A*, 42, 549  
 Hansen, B., 2001, in *ASP. Conf. Ser. Stellar Collisions, Mergers, and their Consequences*, ed. M. Shara (San Francisco: ASP), 400  
 Hartmann, L., 1998, *Accretion Processes in Star Formation* (Cambridge: Cambridge University Press)  
 Hashimoto, M., et al., 1989, 1989, *A&A*, 210, L5  
 Hayashi, C., 1981, *Prog. Theor. Phys. Suppl.*, 70, 35  
 Hernandez, J., et al., 2006, *ApJ*, 652, 472  
 Hubeny, I., 1990, *ApJ*, 351, 632  
 Kalogera, V., 1996 *ApJ*, 471, 352  
 Kenyon, S., Bromley, B., 2006, *AJ*, 131, 1837  
 Konacki, M., Wolszczan, A., 2003, *ApJ*, 589, 495  
 Lin, D.N.C., et al., 1991, *Nature*, 353, 927  
 Livio, M., Pringle, J., Staffer, 1992, *MNRAS*, 257, 15  
 Lynden-Bell, D., Pringle, J., 1974, *MNRAS*, 168, 603  
 Marcy, G., Butler, R. P., 1996, *BAAS*, 27, 1379  
 Mayor, L., Queloz, 1995, *Nature*, 378, 355  
 Menou, K., et al., 2001, *ApJ*, 559, 1032  
 Miller, C., Hamilton, D., 2001, *ApJ*, 550, 863  
 Nomoto, K., Hashimoto, M., 1988, *Phys. Rep.*, 163, 13  
 Phinney, S., Hansen, B., 1993, in *ASP Conf. Ser. 36, Planets around Pulsars*, e.d. J. A. Phillips, S. E. Thorsett, & S. R. Kulkarni (San Francisco: ASP), 371  
 Podsiadlowski, 1993, in *ASP Conf. Ser. 36, Planets around Pulsars*, e.d. J. A. Phillips, S. E. Thorsett, & S. R. Kulkarni (San Francisco: ASP), 371  
 Pollack, J., et al., 1994, *ApJ*, 421, 615  
 Pringle, J., 1981, *ARA&A*, 19, 137  
 Rafikov, R., 2004, *AJ*, 128, 1348  
 Sano, Y., Miyama, S., 1999, *ApJ*, 515, 776  
 Semenov, D., et al., 2003, *A&A*, 410, 611  
 Shakura, N. I. & Sunyaev, R. A. 1973, *A&A*, 24, 337  
 Stepinski, T., Valageas, P., 1997, *A&A*, 319, 1007  
 Wetherill, G., Stewart, G., 1993, *Icar*, 106, 190  
 Wolszczan, A., Frail, 1992, *Nature*, 355, 145  
 Wolszczan, A., 1994, *Science*, 264, 538  
 Wolszczan, A., 2000, *ApJ*, 540, 41  
 Youdin, A., & Chiang, E., 2004, *ApJ*, 601, 1109  
 Youdin, A., & Shu, F., 2002, *ApJ*, 580, 494  
 Zuckerman, B., et al., 1995, *Nature*, 373, 494



TABLE 2  
FREQUENCY-AVERAGED OPACITIES

n	$\kappa_o$	a	b	$T_{max}(K)$
1	$2.0 \times 10^{-4}$	0	2	167
2	$2.0 \times 10^{16}$	0	-7	203
3	$1.0 \times 10^{-1}$	0	0.5	...
4	$2.0 \times 10^{81}$	1	-24	...
5	$1.0 \times 10^{-8}$	2/3	3	...
6	$1.0 \times 10^{-36}$	1/3	10	...
7	$1.5 \times 10^{20}$	1	-5/2	...
8	0.348	0	0	-

NOTE. — Opacity regimes in order of increasing temperature according to the  $\kappa = \kappa_o \rho^a T^b$  power law. The fits are from Bell & Lin(1994). The maximum temperatures for each regime  $n$ , which for  $n \geq 3$  depends on gas density, are found by matching the values for  $\kappa_n$  and  $\kappa_{n+1}$ . The opacity prescription in the n=4-5 regimes is modified as discussed in §3.1.

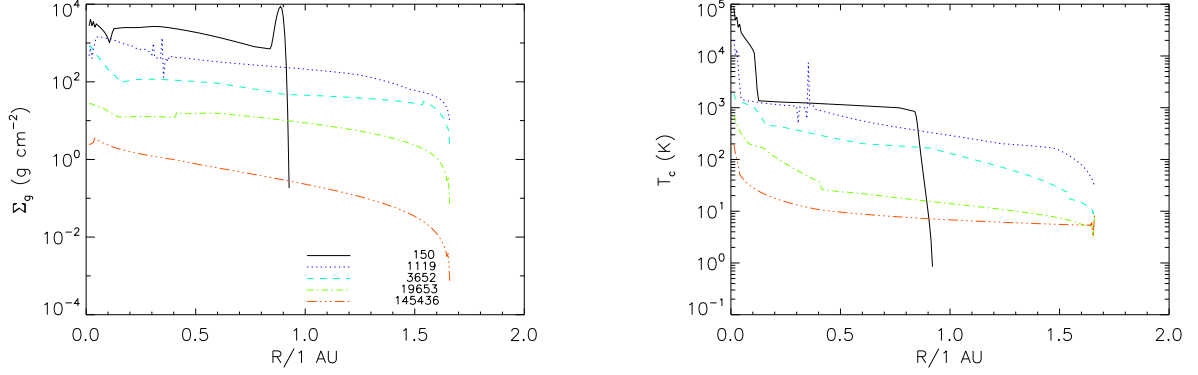


FIG. 1.— Evolution of the gas surface density ( $\Sigma_g$ ) and midplane temperature ( $T_c$ ) profiles for the supernova-fallback disk model, assuming fully viscous accretion. The evolution timescale appears to be  $\tau \sim 10^4$  years, much shorter than typical circumstellar disk evolution timescales ( $\sim 10^6 - 10^7$  years).

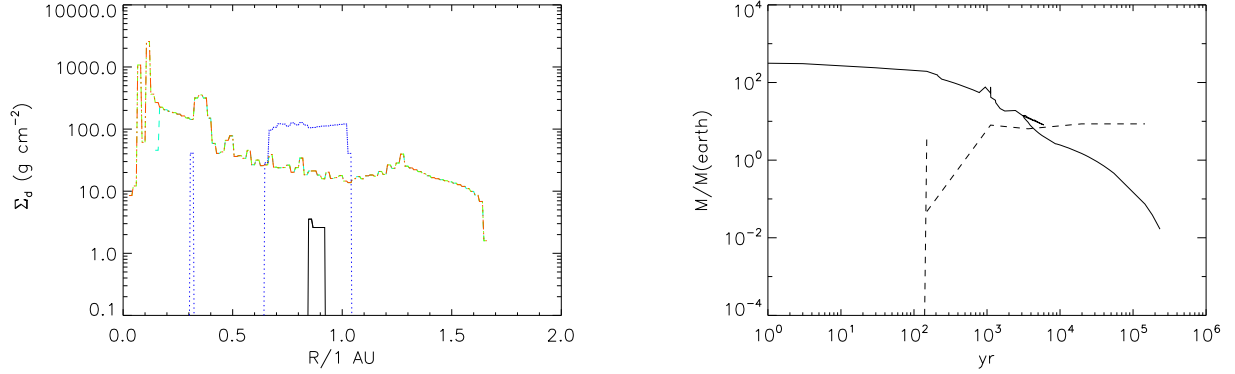


FIG. 2.— (left) Evolution of the solid body surface density ( $\Sigma_d$ ) for the supernova-fallback disk model, assuming fully viscous accretion. Typical  $\Sigma_d$  are  $\sim 10$ - $100\times$  Minimum Mass Solar Nebula (MMSN) values from 0.1-0.5 AU. (right) Mass interior to 1 AU for the supernova-fallback disk model, assuming fully viscous accretion. The gas mass (solid line) drops below  $1 M_{\oplus}$  by  $\sim 10^4$  years. From  $t \sim 10^3$  yr and later, several earth masses of solids (dotted line) reside interior to 1 AU, comparable to the total mass of the three known planets of PSR 1257+12.

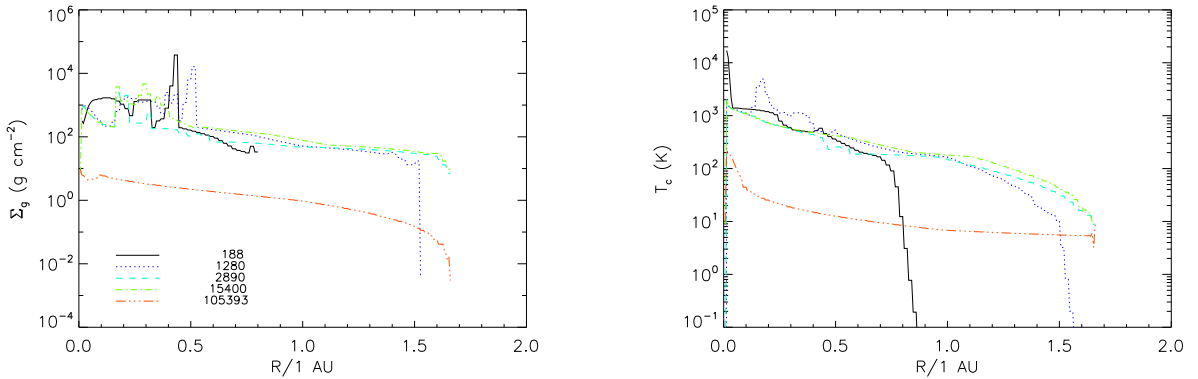


FIG. 3.— Evolution of the gas surface density ( $\Sigma_g$ ) and midplane temperature ( $T_c$ ) profiles for the supernova-fallback disk model, assuming layered accretion. The evolution timescale appears to be slightly longer than the fully viscous case,  $\tau \sim 10^5$ .

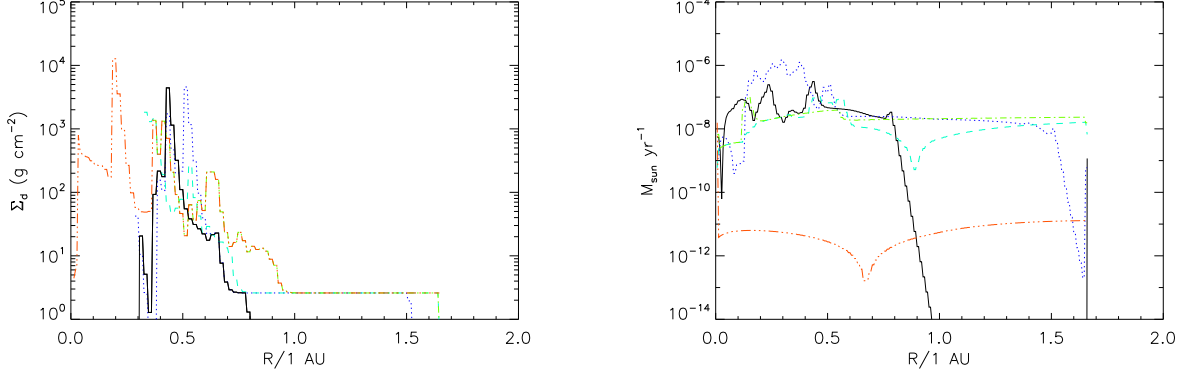


FIG. 4.— Evolution of the solid body surface density ( $\Sigma_d$ ) and accretion rate profiles for the supernova-fallback disk model, assuming layered accretion. A high mass of solids is deposited near the present semimajor axes of the known pulsar planets. The accretion rate shown is the absolute value of the accretion rate: the local minimums reached (e.g. at 0.7 AU for  $t \sim 10^5$  years) occur because the accretion rate changes sign from positive to negative.

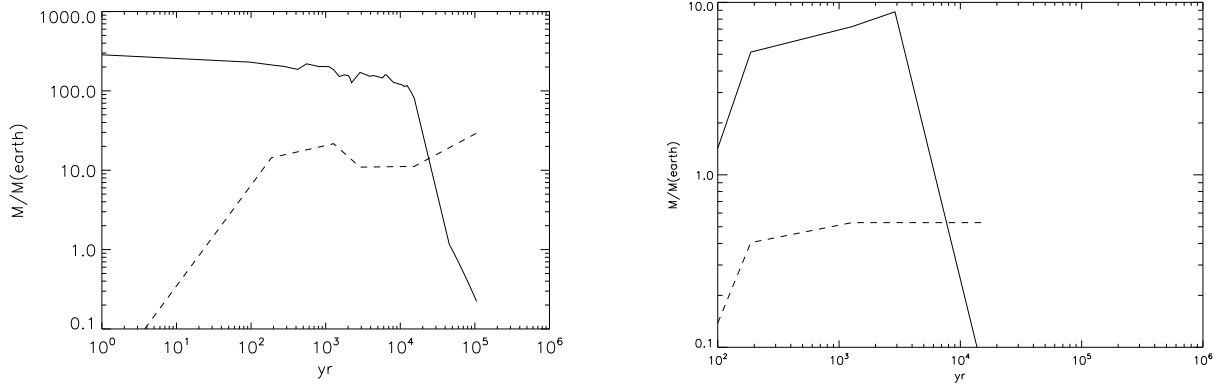


FIG. 5.— Mass interior to 1 AU and exterior to 2 AU for the supernova-fallback disk model, assuming layered accretion. The supernova-fallback model deposits up to  $\sim 30 M_\oplus$  of solid material within 1 AU and  $\sim 0.1\text{--}0.5 M_\oplus$  beyond 2 AU. The likely result is the formation of one or more  $\gtrsim 1 M_\oplus$  bodies interior to 1 AU and perhaps Mars or smaller-sized bodies beyond  $\sim 2$  AU. The formation of gas giant planets is highly unlikely, owing to the fast gas dissipation timescale.

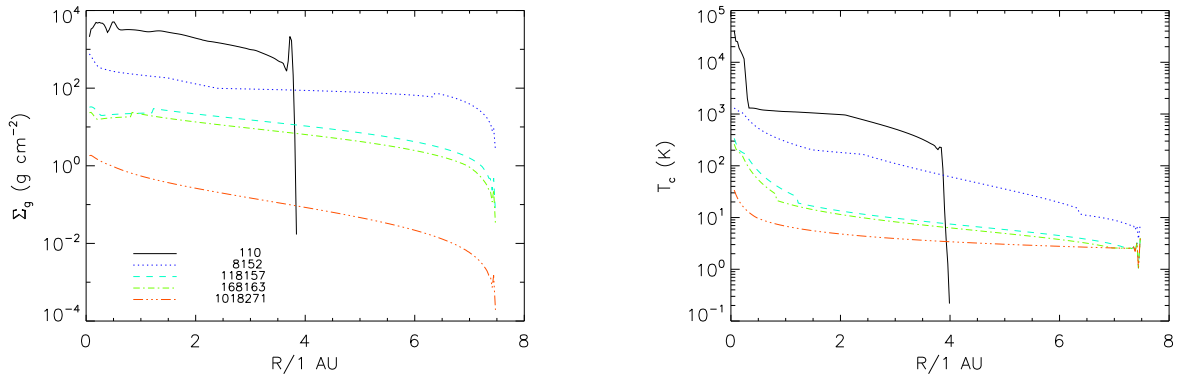


FIG. 6.— Evolution of the gas surface density ( $\Sigma_g$ ) and central temperature profiles for the tidal-disruption disk model, assuming fully viscous accretion and initial angular momentum of  $J \sim 10^{51}$  ergs.s. The disk evolves on  $\tau \sim 10^4$  year timescales.

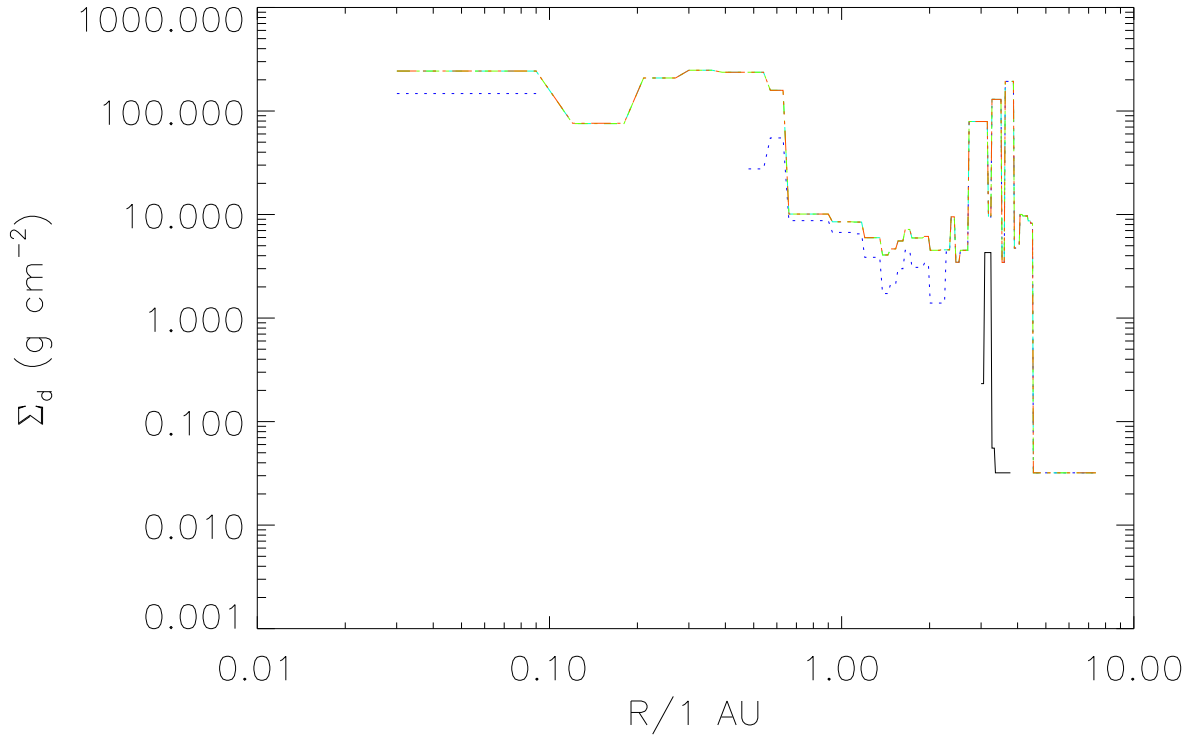


FIG. 7.— Evolution of the solid surface density ( $\Sigma_d$ ) and accretion rate profiles for the tidal-disruption disk model, assuming fully viscous accretion and initial angular momentum of  $J \sim 10^{51}$  ergs.s. Between  $\sim 2$  and 4 AU the disk attains near MMSN values but drops sharply in solid content beyond  $\sim 5$  AU.

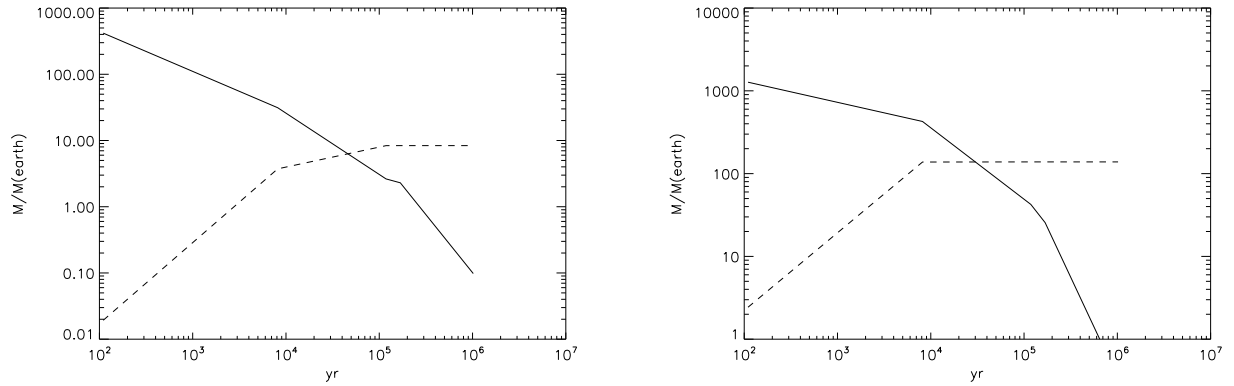


FIG. 8.— The disk mass interior to 1 AU and exterior to 2 AU for the tidal-disruption disk model, assuming fully viscous accretion and initial angular momentum of  $J \sim 10^{51}$  ergs.s. The gas mass  $< 1$  AU drops below  $1M_\oplus$  by  $\sim 2 \times 10^5$  years from an initial value of  $\gtrsim 100M_\oplus$ . The mass beyond 2 AU declines on  $10^4 - 10^5$  year timescales.

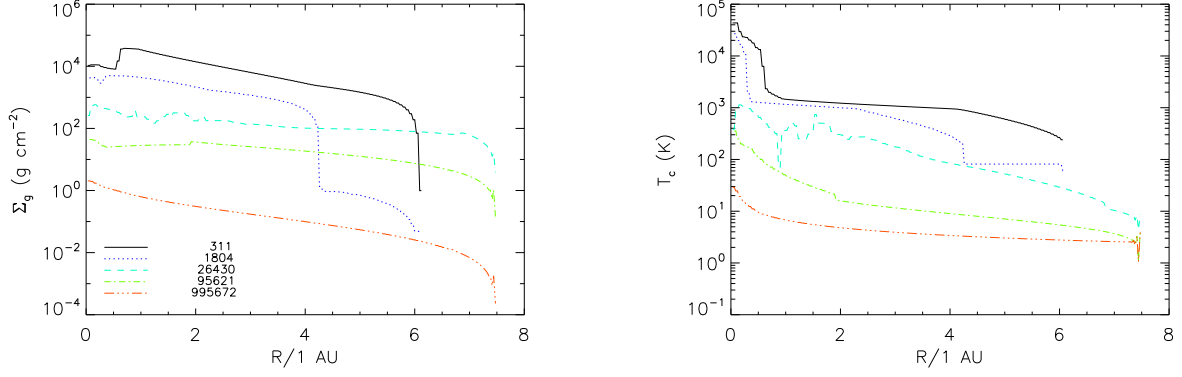


FIG. 9.— Evolution of the gas surface density ( $\Sigma_g$ ) and central temperature profiles for the tidal-disruption disk model, assuming fully viscous accretion and an initial angular momentum of  $J \sim 10^{52}$  ergs.s. The disk evolution is slightly longer but similar to that for the  $J \sim 10^{51}$  model, despite the order-of-magnitude increase in mass.

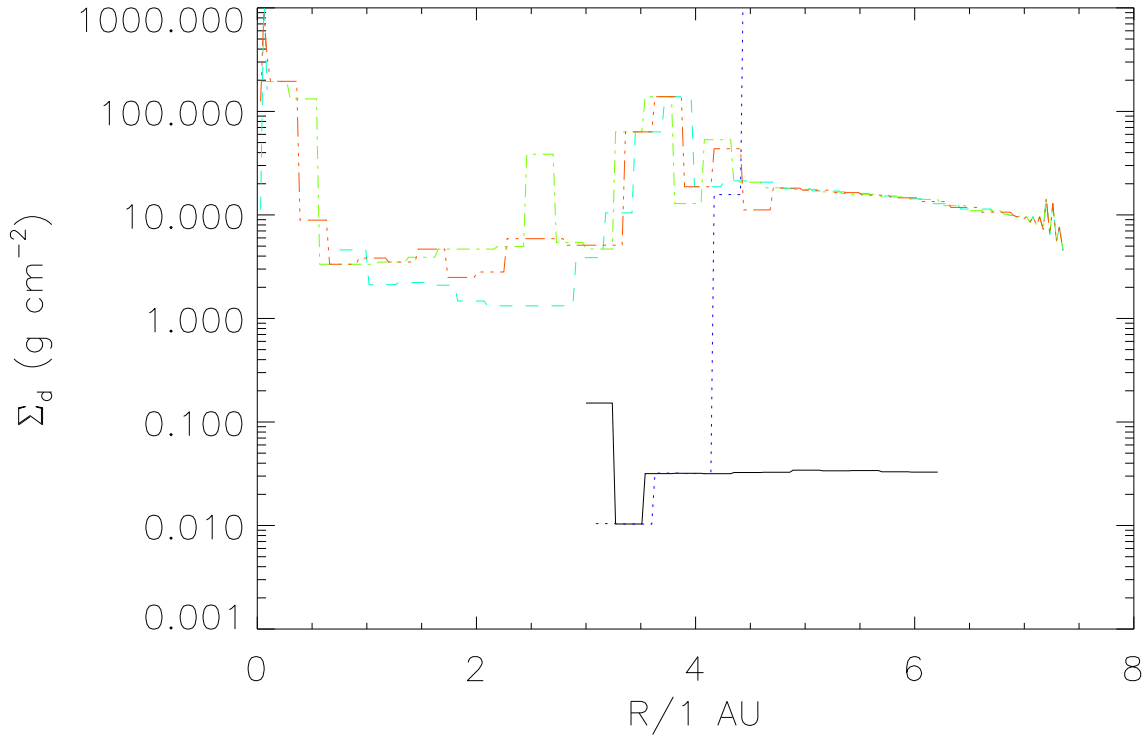


FIG. 10.— Evolution of the solid surface density ( $\Sigma_d$ ) for the tidal-disruption disk model, assuming fully viscous accretion and initial angular momentum of  $J \sim 10^{52}$  ergs.s.  $\Sigma_d$  is slightly larger than MMSN through  $\sim 8 \text{ AU}$ .

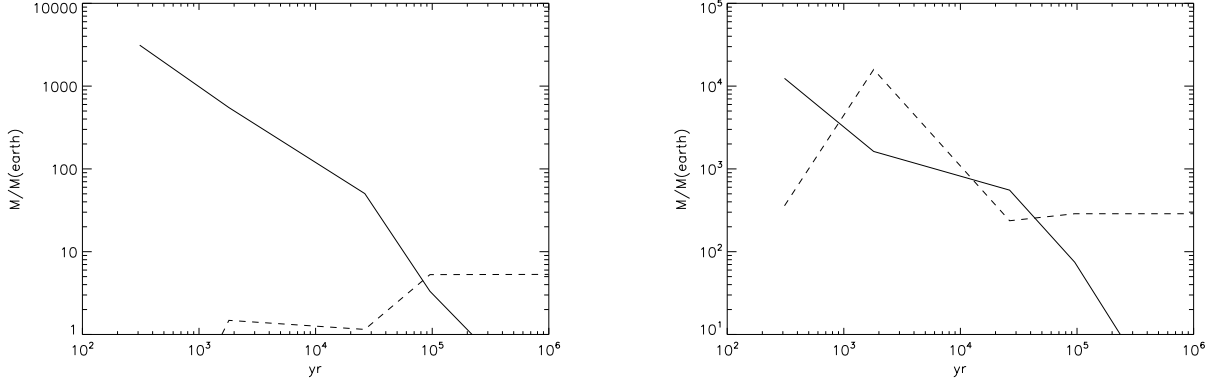


FIG. 11.— The disk mass interior to 1 AU and exterior to 2 AU for the tidal-disruption disk model, assuming fully viscous accretion and initial angular momentum of  $J \sim 10^{52}$  ergs.s. The inner disk does not cool sufficiently to deposit solids until its mass has significantly dropped. The outer ( $\geq 2$  AU) regions of the disk cools fast initially and then heats up again as more material is transported into the outer disk regions. About  $200 M_{\oplus}$  of solids are deposited by  $\sim 10^5$  years. However, the gas mass drops to  $\sim 10 M_{\oplus}$  by  $10^5$  years, making gas giant planet formation highly unlikely.

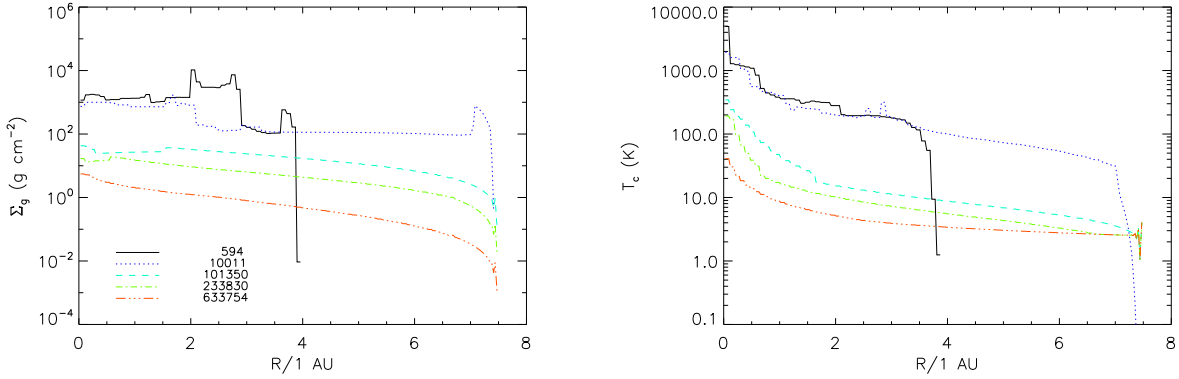


FIG. 12.— Evolution of the gas surface density ( $\Sigma_g$ ) and central temperature profiles for the tidal-disruption disk model, assuming layered accretion and an initial angular momentum of  $J \sim 10^{51}$  ergs.s. The disk assumes a layered state for  $t \lesssim 10^5$  years and the overall evolution timescale is longer than for the fully viscous models and all supernova-fallback models.

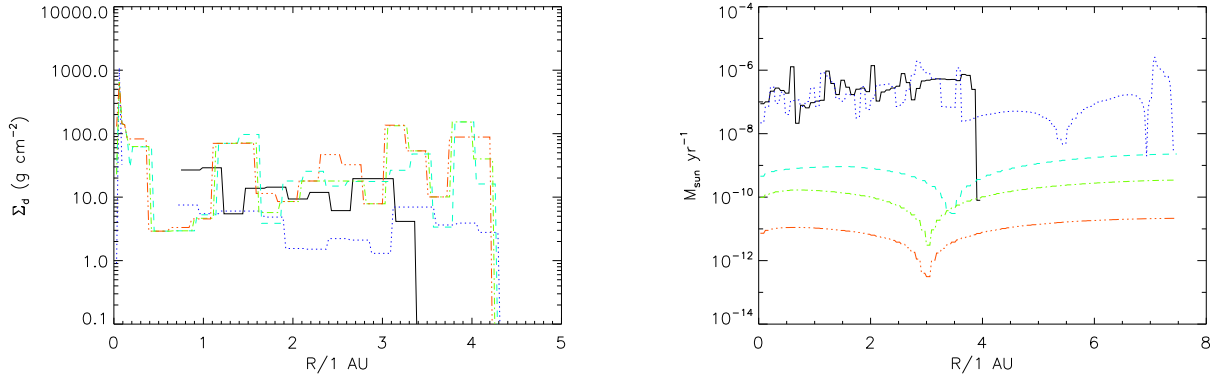


FIG. 13.— Evolution of the solid surface density ( $\Sigma_d$ ) and accretion rate profiles for the tidal-disruption disk model, assuming layered accretion and initial angular momentum of  $J \sim 10^{51}$  ergs.s. A substantial solid column density forms by  $\sim 10^5$  years and fluctuates between 10 and  $100 \text{ g cm}^{-2}$ , with a median value of  $\sim 30\text{--}40 \text{ g cm}^{-2}$ . During the layered accretion state the disk typically has a flat or inverted mass accretion rate profile throughout the disk, consistent with that expected for a layered, episodically accreting disk (Armitage, Livio, & Pringle 2001).

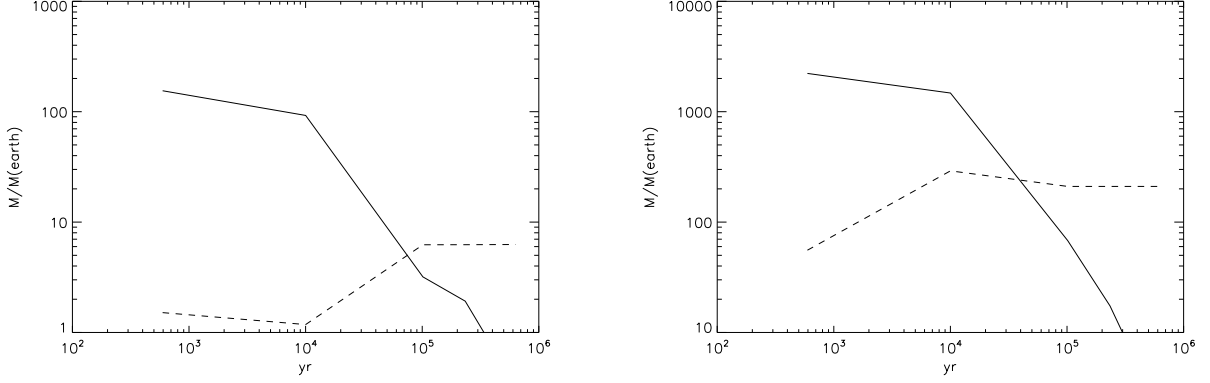


FIG. 14.— The disk mass interior to 1 AU and exterior to 2 AU for the tidal-disruption disk model, assuming layered accretion and an initial angular momentum of  $J \sim 10^{51}$  ergs.s. The final solid mass is reached very fast, within  $2 \times 10^5$  and  $10^4$  years for material  $\leq 1$  and  $\geq 2$  AU, respectively. The disk gas mass drops precipitously after  $10^4$  years.

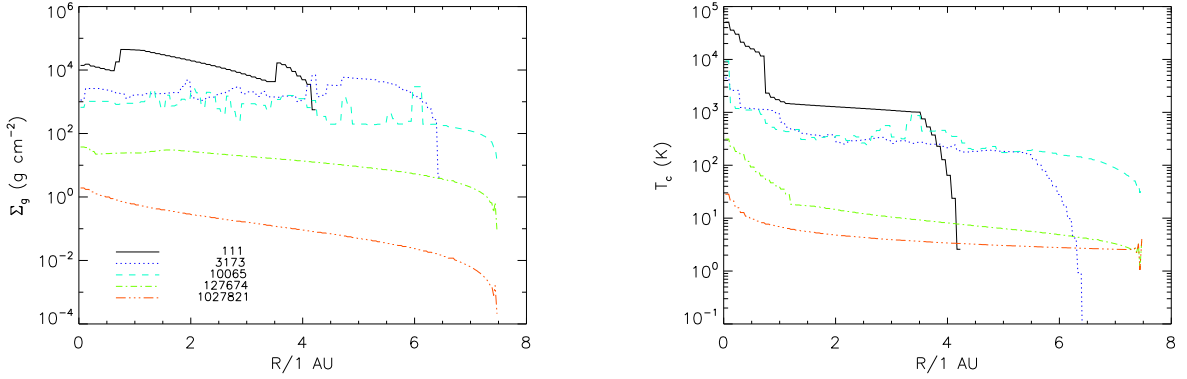


FIG. 15.— Evolution of the gas surface density ( $\Sigma_g$ ) and central temperature profiles for the tidal-disruption disk model, assuming layered accretion and an initial angular momentum of  $J \sim 10^{52}$  ergs.s. The disk stays in a layered state from  $10^3$  to  $10^5$  years and evolves on  $\sim 10^5$  year timescales.

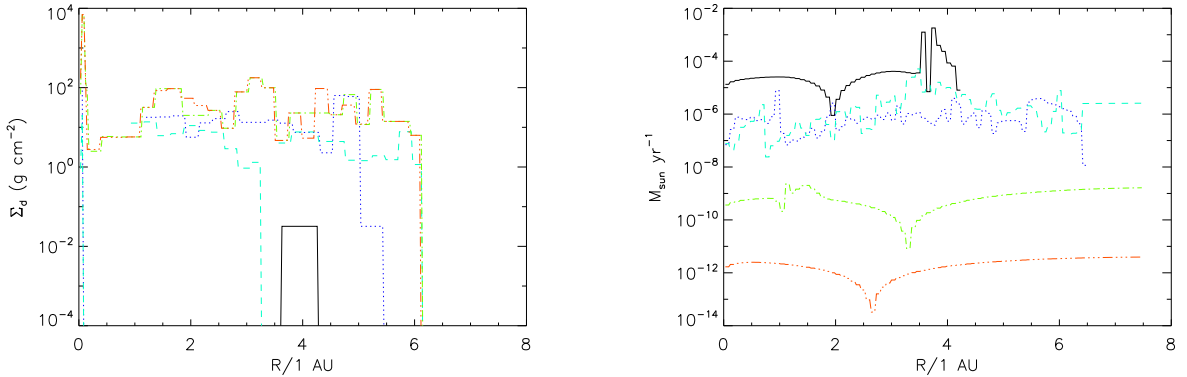


FIG. 16.— Evolution of the solid surface density ( $\Sigma_d$ ) and accretion rate profiles for the tidal-disruption disk model, assuming layered accretion and initial angular momentum of  $J \sim 10^{52}$  ergs.s. The solid surface density ranges from  $\sim 10$ - $100 \text{ g cm}^{-2}$  from 1-6 AU. Less material is deposited beyond 6 AU and from 0.2-1 AU. While the disk is in a layered state, the mass accretion rate also appears to be generally flat or increasing with distance as with the  $J \sim 10^{51}$  case.

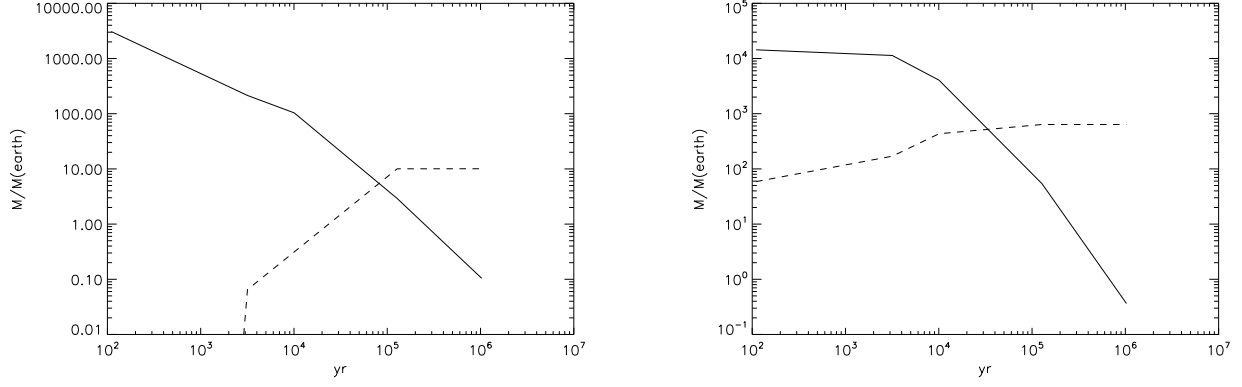


FIG. 17.— The disk mass interior to 1 AU and exterior to 2 AU for the tidal-disruption disk model, assuming layered accretion and an initial angular momentum of  $J \sim 10^{52}$  ergs.s. About 10 and 500  $M_{\odot}$  of solids emerge by  $\sim 10^5$  years interior to 1 AU and exterior to 2 AU, respectively. However, by  $\sim 10^5$  most of the gas has depleted where solids exist ( $\lesssim 6$  AU), making formation of gas giant planets highly unlikely.